Choosing Round Constants in Lightweight Block Ciphers and Cryptographic Permutations

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(joint work with Anne Canteaut, Gregor Leander, and Yann Rotella)

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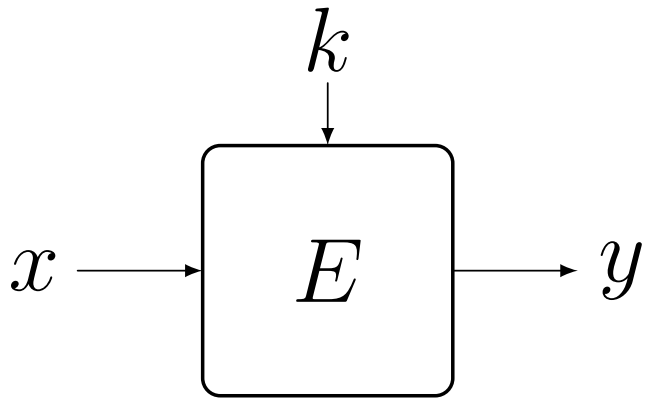
Block Cipher vs. Cryptographic Permutation



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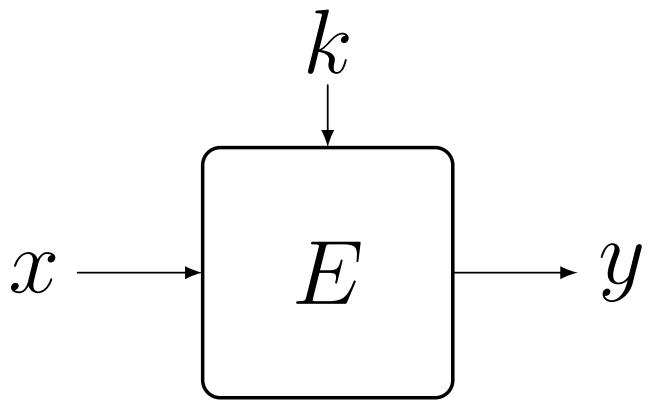

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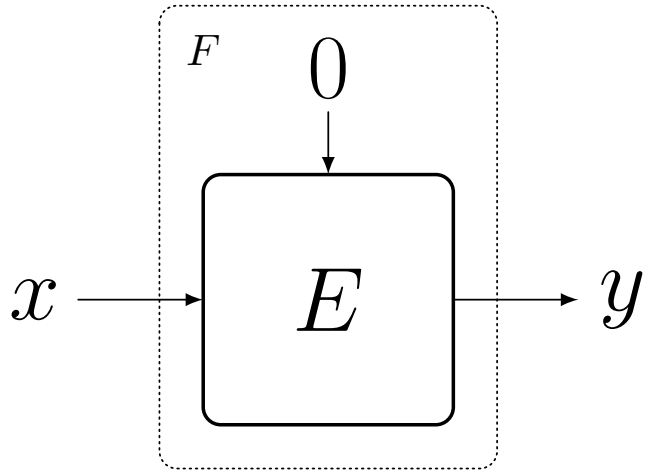

$F\colon \mathbb{F}_2^n \to \mathbb{F}_2^n,\ x \mapsto E_0(x) = y$

Special Case

A cryptographic permutation can be seen as a block cipher with a fixed key!

Why Round Constants are Needed

- Avoiding to use always the same round ($\rightarrow$ slide attacks)
- Avoiding symmetries

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Example: NORX [Aumasson, Jovanovic, Neves 2014]

If the input to the NORX permutation is in the set

$$\mathcal{S} = \left\{ \begin{pmatrix} a & a & a & a \\ b & b & b & b \\ c & c & c & c \\ d & d & d & d \end{pmatrix} \mid a, b, c, d \in \mathbb{F}_2^w \right\},$$

the output also is in $\mathcal{S}$.

Basic Definitions

Definition: Invariant Set

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ be a permutation. We say that $\mathcal{S} \subseteq \mathbb{F}_2^n$ is an invariant set under F if $F(\mathcal{S}) = \mathcal{S}$ or $F(\mathcal{S}) = \mathbb{F}_2^n \setminus \mathcal{S}$.

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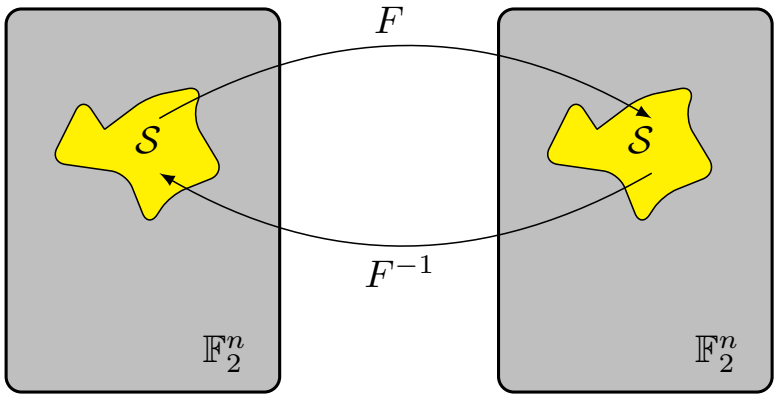

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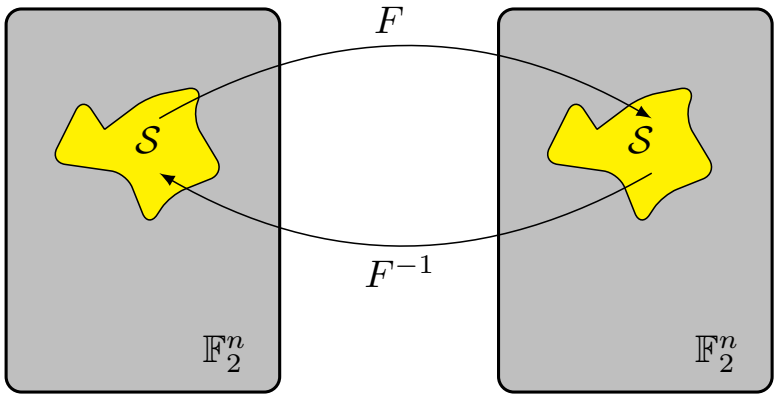

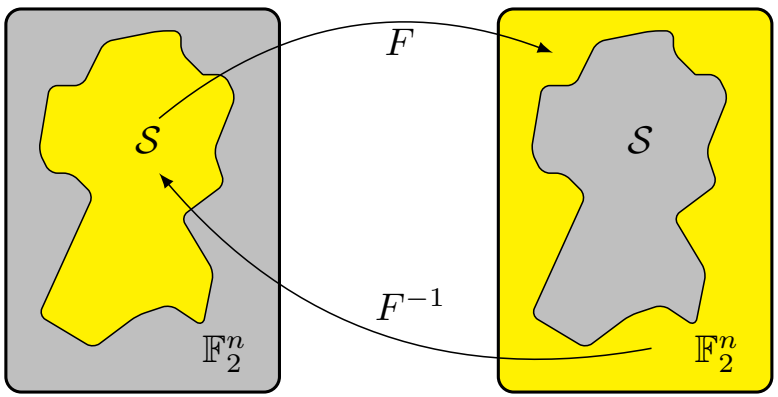

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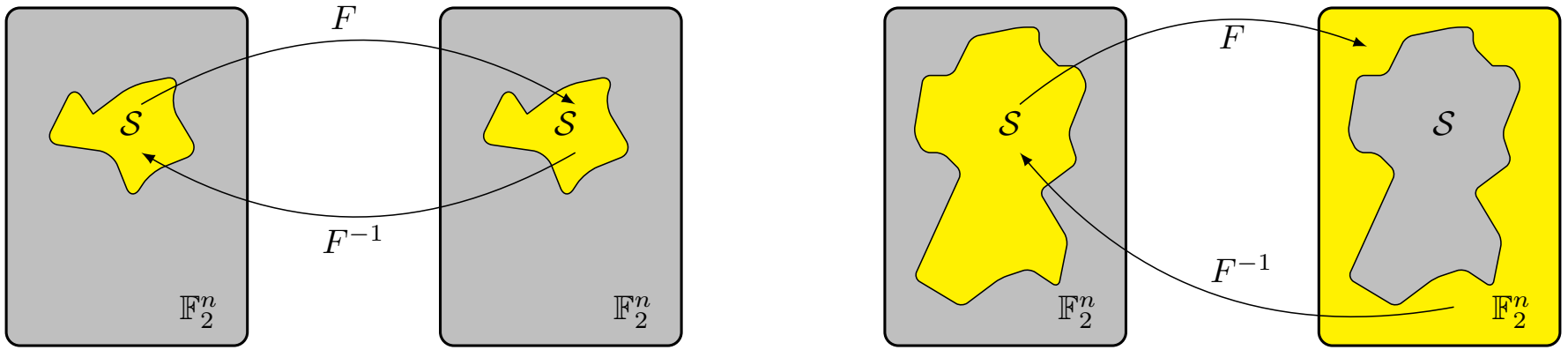

Equivalently:

Let g be the Boolean function defined by $g(x) := 1$ iff $x \in \mathcal{S}$. Then,

$$\forall x \in \mathbb{F}_2^n : g(F(x)) = g(x) \text{ or } \forall x \in \mathbb{F}_2^n : g(F(x)) = g(x) + 1.$$

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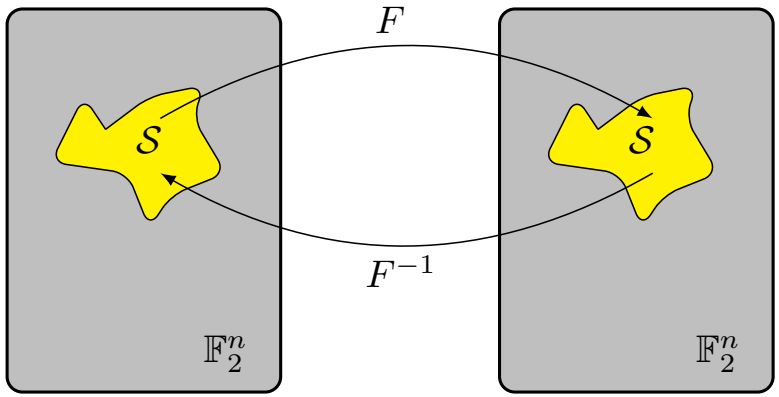

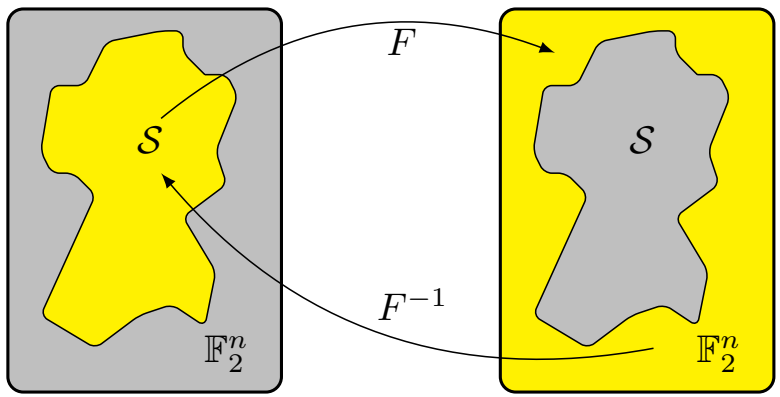

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Definition: Invariant Function

Any Boolean function $g : \mathbb{F}_2^n \to \mathbb{F}_2$ for which $g \circ F + g$ is constant is called an invariant for F.

Invariant Attacks

Examples of invariants g for F: $(g \circ F + g$ is constant)

- $g = \mathbf{0}$. Equivalent to having $\mathcal{S} = \{\}$
- $g = \mathbf{1}$. Equivalent to having $\mathcal{S} = \mathbb{F}_2^n$

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- $g(x) = 1$ iff $x \in \mathcal{U}$ for an affine subspace $\mathcal{U} \subseteq \mathbb{F}_2^n$.
  $\rightarrow$ Invariant Subspace Attack [Leander et al. 2011]

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Consider a block cipher $E : \mathbb{F}_2^n \times \mathbb{F}_2^\kappa \rightarrow \mathbb{F}_2^n,\ (x, k) \mapsto E_k(x)$.

Nonlinear Invariant Attack [Todo, Leander, Sasaki 2016]

If for some keys k, one can find (non-trivial) invariants for E_k, the cipher is vulnerable to the Nonlinear Invariant Attack. Keys which allow for the attack are called weak keys of E.

This Leads to a Distinguisher

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- choose $m_1, \ldots, m_d \in \mathbb{F}_2^n$
- check if $\forall i \in \{1, \ldots, d\} : g(\mathcal{O}(m_i)) + g(m_i)$ is constant

Many Lightweight Ciphers Vulnerable to Invariant Attacks

For instance:

- PRINT-cipher [Leander et al. 2011]
- Midori-64 [Guo et al. 2016] [Todo, Leander, Sasaki 2016]
- iSCREAM [Leander, Minaud, Rønjom 2015]
- SCREAM [Todo, Leander, Sasaki 2016]
- NORX v2.0 [Chaigneau et al. 2017]
- Simpira v1 [Rønjom 2016]
- Haraka v.0 [Jean 2016]

Main Goal: Prevent against Invariant Attacks

Our Main Goal (Block Ciphers)

Given a block cipher $E : \mathbb{F}_2^n \times \mathbb{F}_2^\kappa \to \mathbb{F}_2^n,\ (x, k) \mapsto E_k(x)$. Show that there are no weak keys, i.e., for any k, one can find only trivial invariants for E_k.

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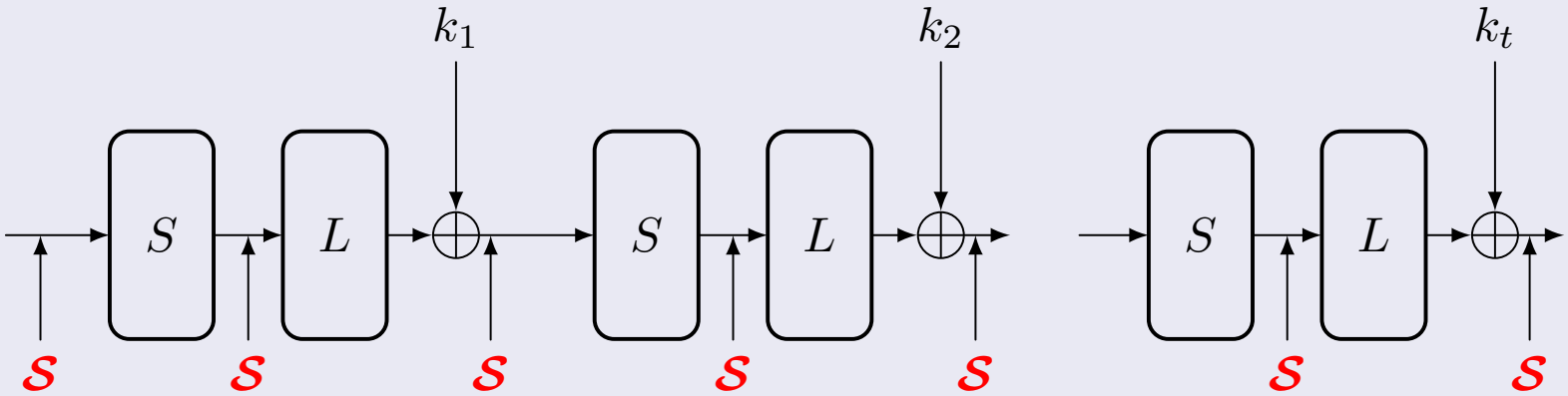
Our Main Goal (Cryptographic Permutation)

Given a permutation $F : \mathbb{F}_2^n \to \mathbb{F}_2^n,\ x \mapsto F(x)$. Show that one can find only trivial invariants for F.

Our Model

Simplification (SPN): Assume the Same Invariant for all Layers

We consider only those invariants g, that are simultaneously invariants for the S-box layer and for all $\mathsf{Add}_{\mathsf{k_i}} \circ L$.



Almost all real attacks we know exploit such an iterative structure!
One exception: [Beyne 2018]

1. Lightweight SPNs: Proving Resistance against Invariant Attacks

2. Design Criteria on the Linear Layer and the Round Constants

Structure of the Invariants for all $\mathrm{Add}_{k_i} \circ L$

Let g be an invariant for both $\mathrm{Add}_{k_i} \circ L$ and $\mathrm{Add}_{k_j} \circ L$. We then have:

$$g(L(x) + k_i) = g(x) + \text{const.} \qquad g(L(x) + k_j) = g(x) + \text{const.}$$

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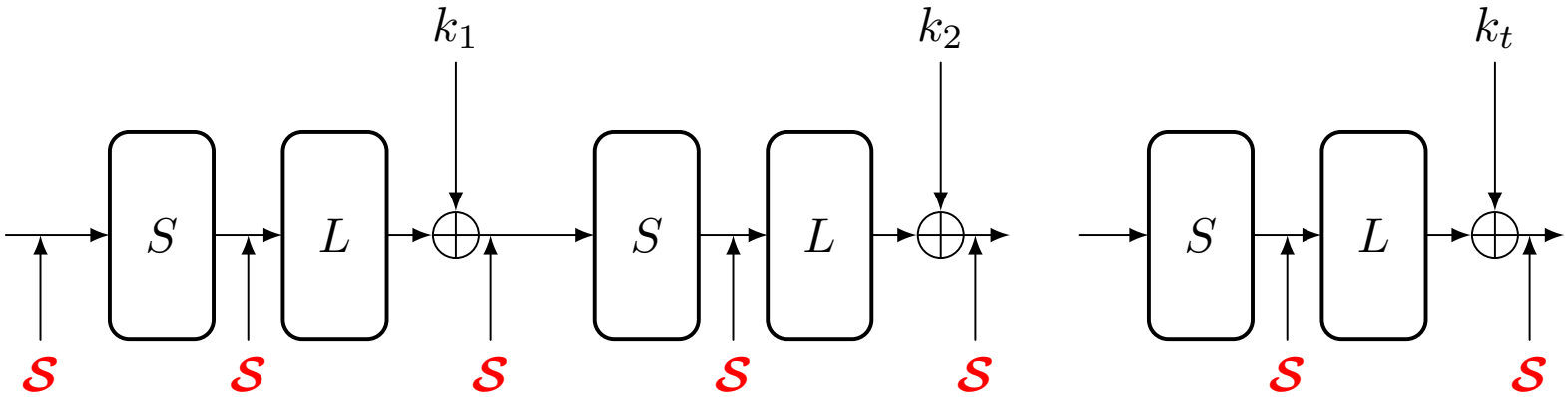
$(\mathbf{k_i} + \mathbf{k_j})$ is a linear structure of g.

Definition: Linear Structures of a Boolean Function g

$$\mathrm{LS}(g) := \{\alpha \in \mathbb{F}_2^n : x \mapsto g(x + \alpha) + g(x) \text{ is constant}\}$$

Requirements on an Invariant g



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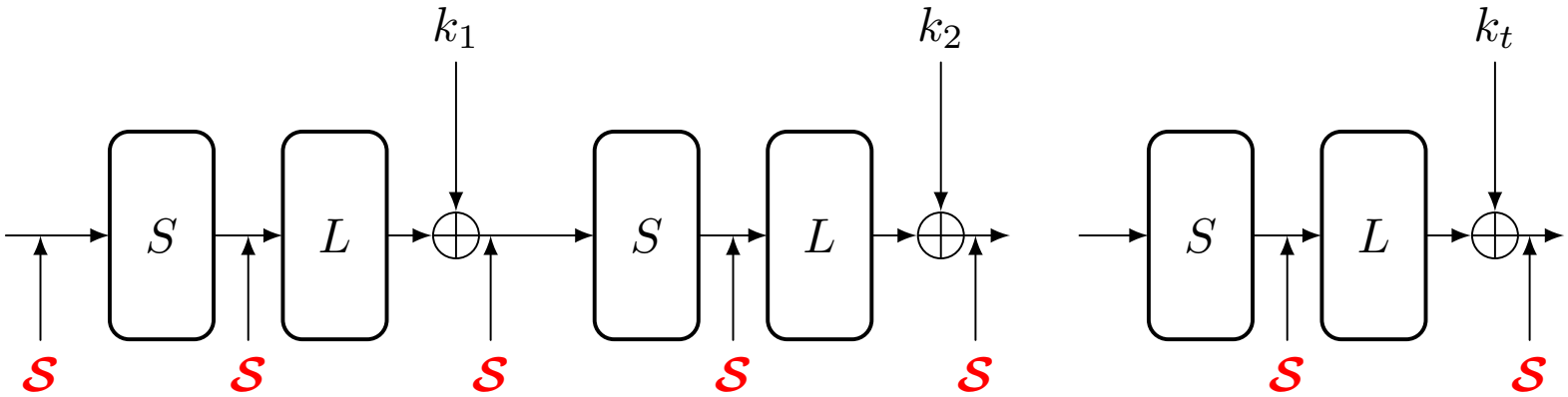

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How does the key schedule look like?

SPNs with very Simple Key Schedules

In many lightweight block ciphers, the round keys only differ by addition of a publicly-known round constant, i.e.,

$$\forall i : k_i := k + c_i.$$

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In a cryptographic permutation, we only have publicly-known round constants, i.e.,

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Proving the Non-Existence of Invariants

Main Condition on an Invariant g

1. g has to be an invariant for the S-box layer and
2. The smallest L-invariant subspace of $\mathbb{F}_2^n$ that contains all $c_i + c_j$ must be a subset of $\mathrm{LS}(g)$. We denote this subspace by $W_L(\{c_i + c_j \mid i, j\})$.

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Assume that the S-box layer has no component of algebraic degree 1.
If $\dim W_L(\{c_i + c_j \mid i, j\}) \geq n - 1$, there are only the trivial invariants that fulfill the above main condition!

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- But then, g is linear (or affine).
- Since the S-box layer does not have a linear (or affine) component, g must be trivial.

Applying the Argument to some Examples

Important Observation

If $\dim W_L(\{c_i + c_j \mid i, j\}) \geq n - 1$, the invariant attack does not apply!

This holds for any (reasonable) choice of the S-box layer.

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How to Use Properties of the S-box

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$$\mathsf{LS}_0(g) := \{\alpha \in \mathbb{F}_2^n : x \mapsto g(x+\alpha) + g(x) = 0\} \subseteq \mathsf{LS}(g)$$

We know $\dim \mathsf{LS}_0(g) \in \{\dim \mathsf{LS}(g), \dim \mathsf{LS}(g) - 1\}$. How to find $\mathsf{LS}_0(g)$?

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Let g be an invariant for $\mathsf{Add}_{\mathsf{k_i}} \circ L$ for some k_i and let V be an L-invariant subspace of $\mathsf{LS}(g)$. Then, for any $s \in V$, it is $s + L(s) \in \mathsf{LS}_0(g)$.

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Second Lemma

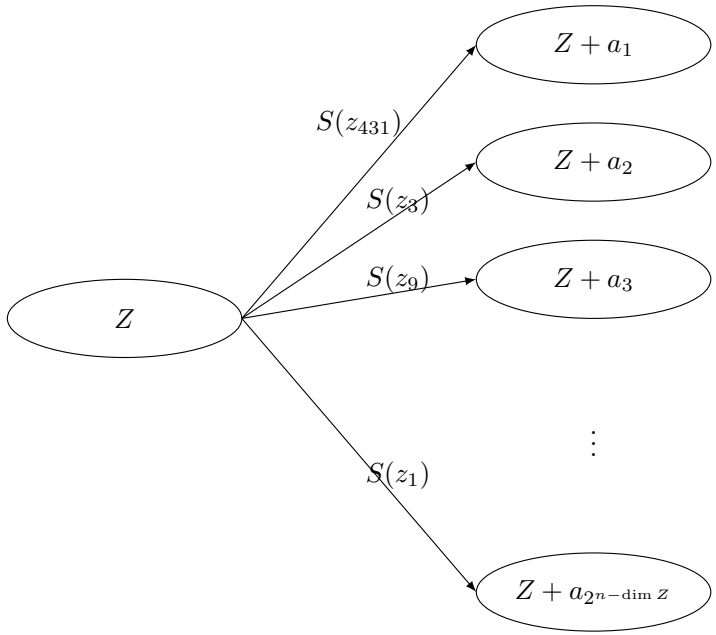
Let g be an invariant for S, where $S \colon \mathbb{F}_2^n \to \mathbb{F}_2^n$ is a permutation with an odd cycle. Then, $s \in \mathsf{LS}(g) \cap \{S(x) + x \mid x \in \mathbb{F}_2^n\}$ implies $s \in \mathsf{LS}_0(g)$.

How to Use Properties of the S-box

Lemma

Let $g\colon \mathbb{F}_2^n \to \mathbb{F}_2$ be an invariant for S and let Z be a subspace of $\mathbb{F}_2^n$ with $Z \subset \mathsf{LS}_0(g)$. Then

- g is constant on each coset $Z + a$
- g is constant on $S(Z)$



How to Use Properties of the S-box

Lemma

Let $g\colon \mathbb{F}_2^n \to \mathbb{F}_2$ be an invariant for S and let Z be a subspace of $\mathbb{F}_2^n$ with $Z \subset \mathrm{LS}_0(g)$. Then

- g is constant on each coset $Z + a$
- g is constant on $S(Z)$

Algorithm

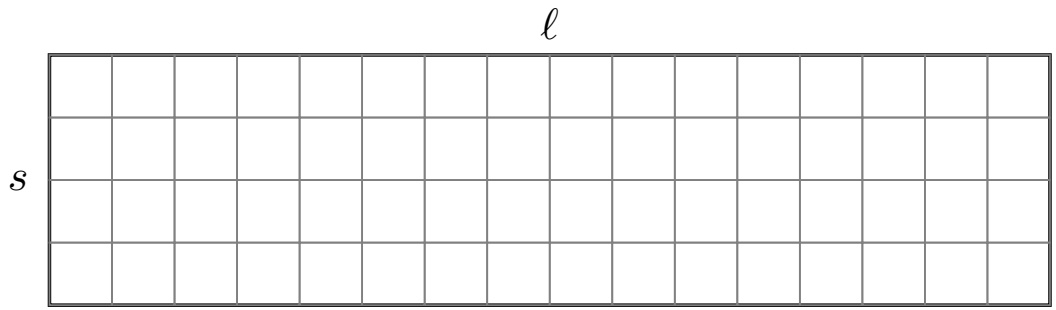
1: $R = \{\}$
2: **repeat**
3: $\quad z \xleftarrow{\$} Z$
4: $\quad$ Compute $S(z)$
5: $\quad$ Add to R a representative of the coset defined by $S(z)$
6: **until** $|R| = 2^{n-\dim Z}$

Example: LS Designs (e.g., Ascon)

Important Observation

If $\dim W_L(\{c_i + c_j \mid i, j\}) \geq n - 1$, the invariant attack does not apply!

This holds for any (reasonable) choice of the S-box layer.



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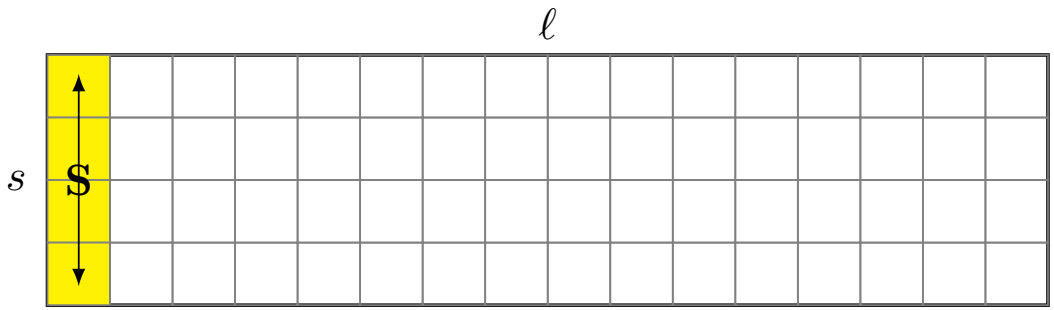

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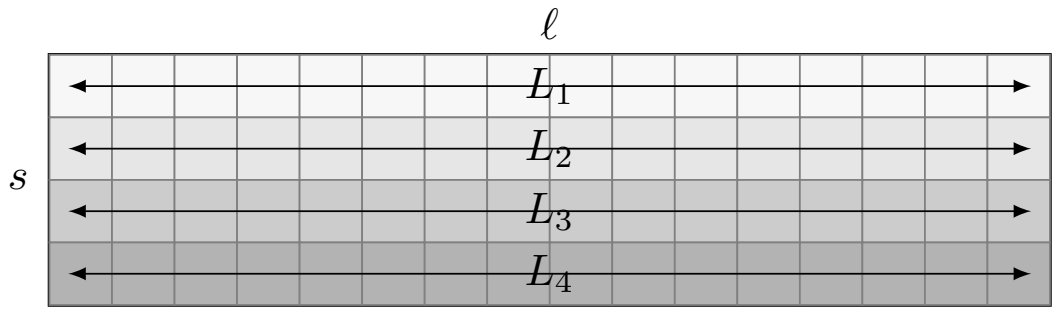

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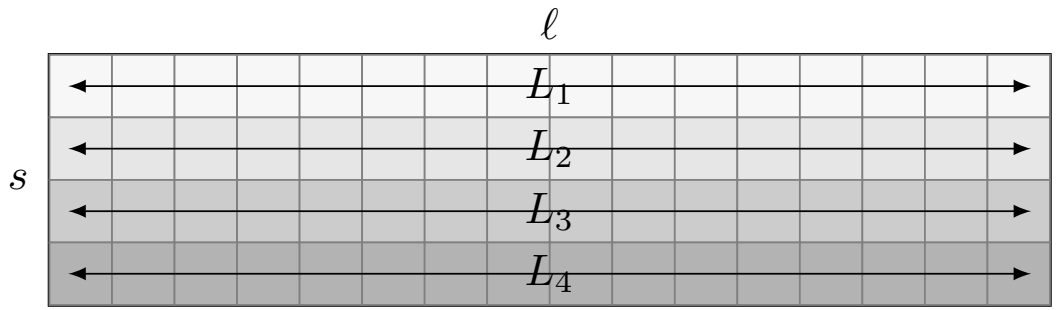

$$L = \begin{pmatrix} L_1 & & & \\ & L_2 & & \\ & & L_3 & \\ & & & L_4 \end{pmatrix}$$

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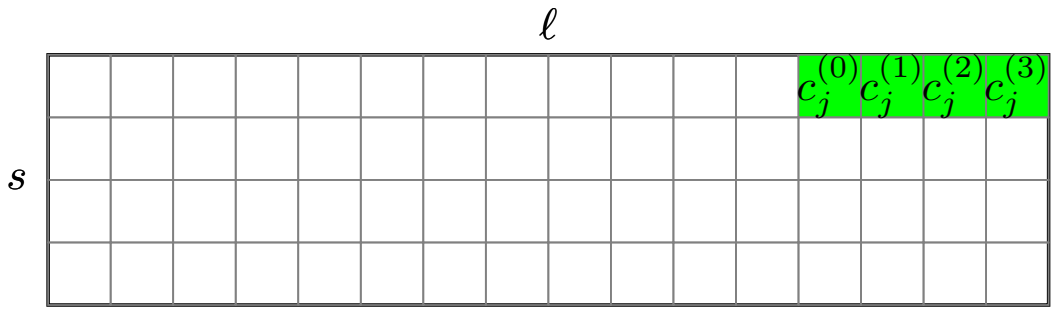

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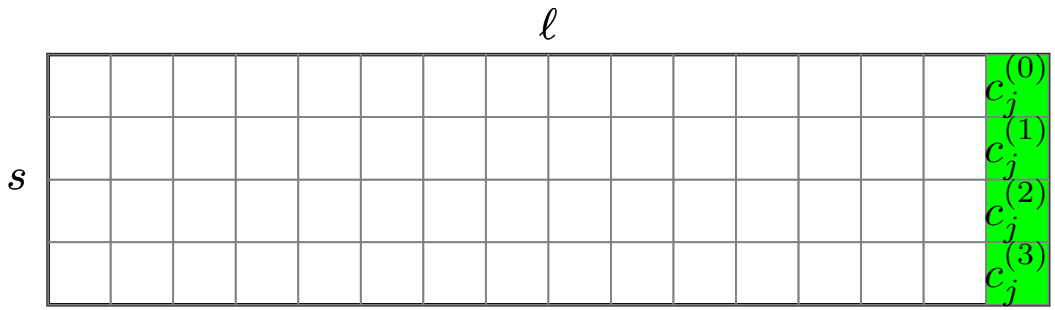

For the argument, it would be better to put the constants in the columns.

Remark

A low dimension doesn't imply insecurity of the permutation!

1 Lightweight SPNs: Proving Resistance against Invariant Attacks

2 Design Criteria on the Linear Layer and the Round Constants

Very Different Behavior for each Cipher

Skinny-64-64. $\dim W_L(\{c_i + c_j \mid i, j\}) = 64$

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Are the constants for Prince and Mantis just unluckily chosen?

It's a Property of the Linear Layer

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There exists a $c \in \mathbb{F}_2^n$ such that $\dim W_L(c) = d$ if and only if d is the degree of a divisor of the minimal polynomial m_L of L.

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$$\Rightarrow \max_{c \in \mathbb{F}_2^n} \dim W_L(c) = \deg m_L$$

Examples

Skinny-64.

$$m_L = (X+1)^{16} \in \mathbb{F}_2[X]$$

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Mantis and Midori.

$$m_L = (X+1)^6 \in \mathbb{F}_2[X]$$

There exists a $c \in \mathbb{F}_2^{64}$ with $\dim W_L(c) = d$ if and only if $d \in \{1, \ldots, 6\}$.

Considering more Constants: The Rational Canonical Form

If $\deg(m_L) = n$, there exists a basis for which the matrix of L is the **companion matrix** of m_L.

Definition: Companion Matrix

Let $p = X^m + \prod_{i=0}^{m-1} p_i X^i \in \mathbb{F}_2[X]$. The companion matrix of p is

$$C(p) := \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ p_0 & p_1 & p_2 & \dots & p_{m-1} \end{pmatrix}$$

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In general, there exists a basis for which the matrix of L is

$$\begin{pmatrix} C(Q_1) & & & \\ & C(Q_2) & & \\ & & \ddots & \\ & & & C(Q_r) \end{pmatrix}$$

for r polynomials $Q_r \mid Q_{r-1} \mid \cdots \mid Q_1 = m_L$.
$Q_1, Q_2, \dots, Q_r$ are called the **invariant factors of L**.

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Theorem

Let $Q_1, Q_2, \ldots, Q_r$ be the invariant factors of L. For any $t \leq r$, we have

$$\max_{c_1,\ldots,c_t} \dim W_L(\{c_1, \ldots, c_t\}) = \sum_{i=1}^{t} \deg Q_i.$$

In particular, one needs r elements to obtain the maximal dimension $\mathbb{F}_2^n$.

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Prince. The invariant factor decomposition is

$$\begin{aligned} Q_1 &= Q_2 \\ &= X^{20} + X^{18} + X^{16} + X^{14} + X^{12} + X^8 + X^6 + X^4 + X^2 + 1 \\ Q_3 &= Q_4 = X^8 + X^6 + X^2 + 1 \\ Q_5 &= Q_6 = Q_7 = Q_8 = X^2 + 1 \end{aligned}$$

For $t = 5$, $\max \dim W_L(\{c_1, \ldots, c_5\}) = 20 + 20 + 8 + 8 + 2 = 58$.
We need **8 elements** to get the full space.

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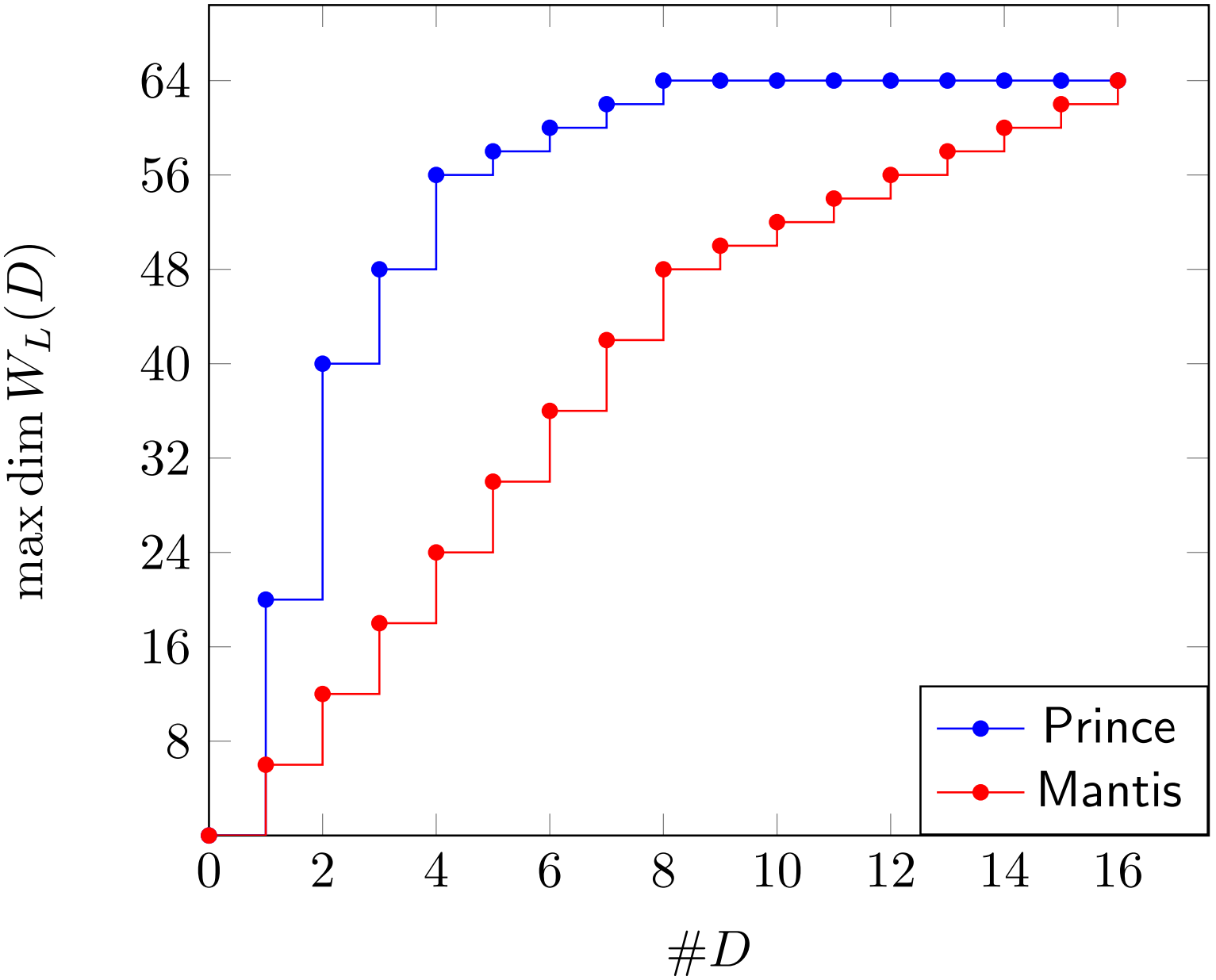
$$\begin{aligned} Q_1 &= Q_2 = Q_3 = Q_4 = Q_5 = Q_6 = Q_7 = Q_8 = X^6 + 1 \\ Q_9 &= Q_{10} = Q_{11} = Q_{12} = Q_{13} = Q_{14} = Q_{15} = Q_{16} = X^2 + 1 \end{aligned}$$

For $t = 7$, $\max \dim W_L(\{c_1, \ldots, c_7\}) = 42$.
For $t = 8$, $\max \dim W_L(\{c_1, \ldots, c_8\}) = 48$.
We need **16 elements** to get the full space.

The Maximal Dimension for $\#D$ Constants



Choosing Random Round Constants

For $t \geq r$, the probability that t uniformly chosen constants c_i generate the whole $\mathbb{F}_2^n$ can be computed from the invariant factors of L.

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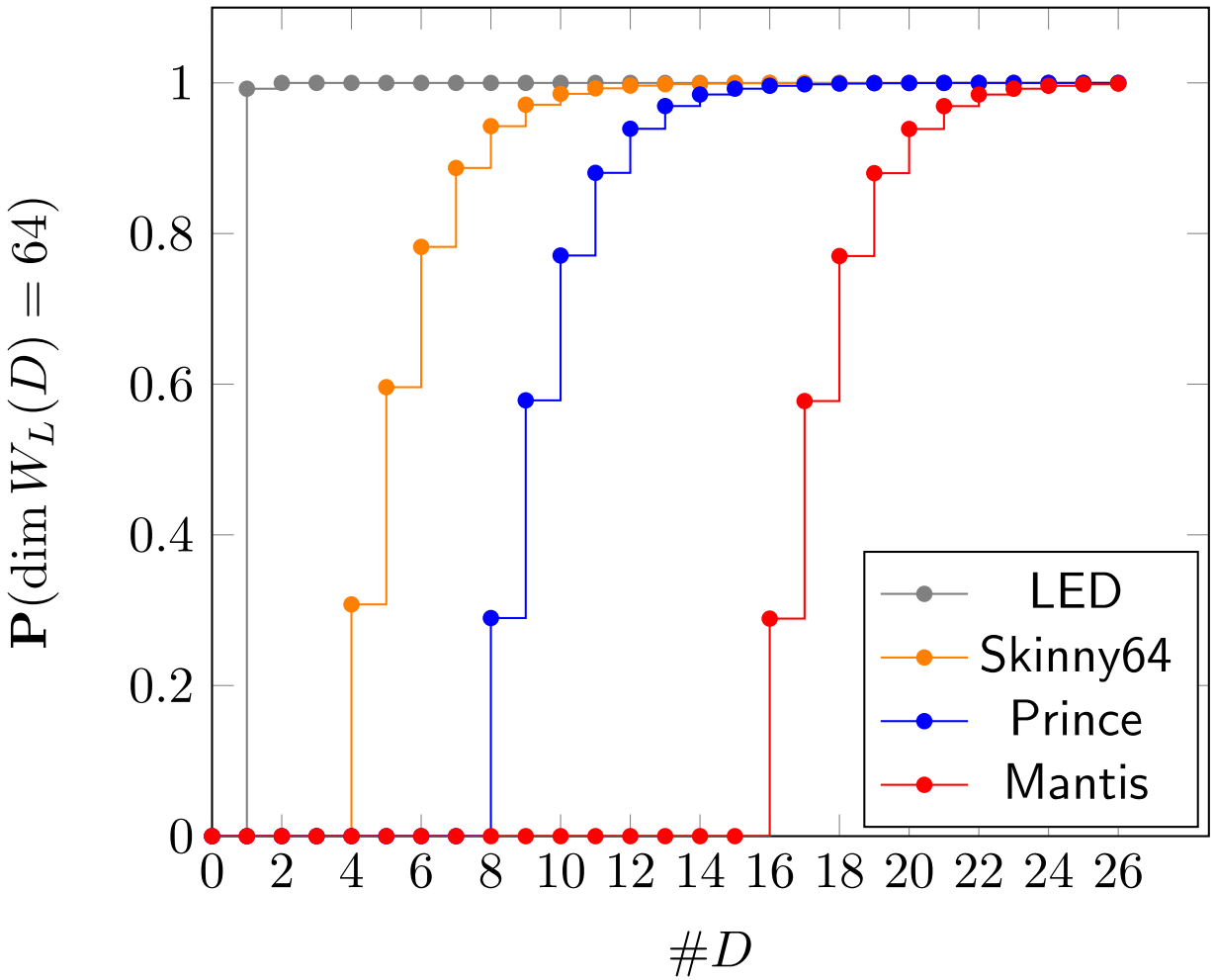

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- **Input:** Linear layer L, RC differences $\{c_i + c_j \mid i, j\}$
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Thanks for your attention! Any questions?